Example 8.1: A 3-phase single circuit transmission line is 400 km long. If the line is rated for 220 kV and has the parameters, $R = 0.1$ ohms/km, $L = 1.26$ mH/km, $C = 0.009$ μF/km, and $G = 0$, find (*a*) the surge impedance and (*b*) the velocity of propagation neglecting the resistance of the line. If a surge of 150 kV and infinitely long tail strikes at one end of the line, what is the time taken for the surge to travel to the other end of the line ?

Solution : Velocity of propagation $= \dfrac{1}{\sqrt{LC}}$

$$= \frac{1}{\sqrt{1.26 \times 10^{-3} \times 0.009 \times 10^{-6}}}$$

$$= 3 \times 10^5 \text{ km/s}$$

Surge impedance $= \sqrt{\dfrac{L}{C}}$

$$= \sqrt{\frac{1.26 \times 10^{-3}}{9 \times 10^{-9}}}$$

$$= 374.2\ \Omega$$

Time taken for the surge to travel to the other end is

$$= \frac{400}{3 \times 10^5}$$

$$= 1.33 \text{ m s}$$

Example 8.2: A transmission line of surge impedance 500 Ω is connected to a cable of surge impedance 60 Ω at the other end. If a surge of 500 kV travels along the line to the junction point, find the voltage build-up at the junction ?

Solution:

$$e = 500\, U(t) \text{ kV}$$

$$Z_1 = 500\ \Omega$$

$$Z_2 = 60\ \Omega$$

Coefficient of reflection,

$$\Gamma = \frac{(Z_2 - Z_1)}{(Z_2 + Z_1)} = \frac{(500 - 60)}{(500 + 60)}$$

$$= 0.786$$

Magnitude of the transmitted wave to the cable

$$= (1 + \Gamma)\, e$$

$$= (1.786) \times 500$$

$$= 893 \text{ kV}$$

= junction voltage

Example 8.3: An infinite rectangular wave on a line having a surge impedance of 500 Ω strikes a transmission line terminated with a capacitance of 0.004 μF. Calculate the extent to which the wave front is retarded ?

Solution: $e = E\,U(t)$, $Z_1 = 500\ \Omega$, and $Z_2 = \dfrac{1}{Cs} = \dfrac{10^9}{4s}$

$$\Gamma = \frac{Z_2 - Z_1}{Z_2 + Z_1}$$

$$= \frac{\left(\dfrac{10^9}{4s} - 500\right)}{\left(\dfrac{10^9}{4s} + 500\right)}$$

$$= \frac{(10^9 - 2000s)}{(10^9 + 2000s)}$$

$$(1 + \Gamma) = \frac{2 \times 10^9}{(10^9 + 2000\,s)}$$

$$= \frac{10^6}{\left(\dfrac{10^6}{2} + s\right)}$$

$e''(s)$ voltage across capacitor $= (1 + \Gamma)\, e(s)$

$$= \frac{10^6}{\left(\dfrac{10^6}{2} + s\right)} \frac{E}{s}$$

Taking inverse transforms

$$e'' = 2E\left[1 - \exp\left(-\frac{t}{2 \times 10^{-6}}\right)\right]$$

Taking the rise time to be $3CZ$, the wave is retarded by

$$3 \times 2 \times 10^{-6} = 6 \times 10^{-6}\ \text{s}$$

or by 6 μ s.

Example 8.5: An underground cable of inductance 0.189 mH/km and of capacitance 0.3 μF/km is connected to an overhead line having an inductance of 1.26 mH/km and capacitance of 0.009 μF/km. Calculate the transmitted and reflected voltage and current waves at the junction, if a surge of 200 kV travels to the junction, (*i*) along the cable, and (*ii*) along the overhead line.

Solution: Surge impedance of the cable $Z_1 = \sqrt{\dfrac{L_1}{C_1}}$

$$= \sqrt{\frac{0.189\times 10^{-3}}{0.3\times 10^{-6}}}$$

$$= 25.1\ \Omega$$

Surge impedance of the line $Z_2 = \sqrt{\dfrac{L_2}{C_2}}$

$$= \sqrt{\frac{1.26\times 10^{-3}}{0.009\times 10^{-6}}}$$

$$= 374.2\ \Omega$$

When the surge travels along the cable:

$$\Gamma = \frac{(374.2-25.1)}{(374.2+25.1)}$$

$$= 0.8742$$

The reflected wave, $e' = \Gamma e = 0.8742\times 200\ \text{kV}$

$$= 174.84\ \text{kV}$$

The transmitted wave, $e'' = (1+\Gamma)\, e$

$$= 1.8742\times 200\ \text{kV}$$

$$= 374.84\ \text{kV}$$

The reflected current wave, $I' = \dfrac{e'}{Z_1} = \dfrac{174.84\times 10^3}{25.1}$

$$= 6.97\ \text{kA}$$

The transmitted current wave, $I'' = \dfrac{e''}{Z_2} = \dfrac{374.84\times 10^3}{374.20}$

$$= 1.002\ \text{kA}$$

When the wave travels along the line:

$$\Gamma = \frac{(25.1 - 374.2)}{(25.1 + 374.2)}$$

$$= -0.8742$$

$\therefore$ reflected wave $= e' = \Gamma\, e = -174.84$ kV

The transmitted wave $= e'' = (1 + \Gamma)\, e = (1 - 0.8742) \times 200$

$$= 25.16 \text{ kV}$$

The transmitted current wave $= I''$

$$= \frac{e''}{Z_2} = \frac{25.16}{25.1}$$

$$= 1.006 \text{ kA}$$

The reflected current wave $= I'$

$$= \frac{e'}{Z_1} = \frac{-174.84}{374.2}$$

$$= 0.467 \text{ kA or } 467 \text{ A}$$